Compound Interest Variables

- \( P \) = present single sum of money (single cash flow).
- \( F \) = future single sum of money (single cash flow).
- \( A \) = uniform series of money (multiple cash flows).
- \( n \) = number of compounding periods (months, years, etc.)
- \( i \) = period compound interest rate,
- \( i^* \) = investor’s minimum rate of return

Variable Relationships

<table>
<thead>
<tr>
<th>Desired Quantity</th>
<th>Given Quantity</th>
<th>X</th>
<th>Appropriate Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>( P )</td>
<td>( X )</td>
<td>( F/P_{i,n} )</td>
</tr>
<tr>
<td>( F )</td>
<td>( A )</td>
<td>( X )</td>
<td>( F/A_{i,n} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( F )</td>
<td>( X )</td>
<td>( P/F_{i,n} )</td>
</tr>
<tr>
<td>( P )</td>
<td>( A )</td>
<td>( X )</td>
<td>( P/A_{i,n} )</td>
</tr>
<tr>
<td>( A )</td>
<td>( F )</td>
<td>( X )</td>
<td>( A/F_{i,n} )</td>
</tr>
<tr>
<td>( A )</td>
<td>( P )</td>
<td>( X )</td>
<td>( A/P_{i,n} )</td>
</tr>
<tr>
<td>( A )</td>
<td>( G )</td>
<td>( X )</td>
<td>( A/G_{i,n} )</td>
</tr>
</tbody>
</table>
Factor Symbolism

- First, note that with the factor symbolism, there is always an alternating letter symbol approach. For example, we calculate "A" given "F" using an "A/F" factor.
- Second, the first letter in each factor describes what is being calculated.
- Third, think of the "/" as representing the word "given" to better understand which factor to use when.

Example 2-1 Single Payment Compound-Amount Factor Illustration

Calculate the future worth that $1,000 today will have six years from now if interest is 10% per year compounded annually.

\[ P = \$1,000 \]
\[ F = ? \]
\[ i = 10\% \text{ per year} \]
\[ 0 \quad 1 \quad 2 \quad \ldots \quad 6 \]

Solution, Future Balance, \( F = P(1+i)^n = 1,000(1.1)^6 = 1,771.56, \) or:

\[ F = \$1,000(1.1)^6 = 1,771.56 \]
Example 2-1  Single Payment Compound-Amount Factor Illustration

Calculate the future worth that $1,000 today will have six years from now if interest is 10% per year compounded annually.

\[ P = \$1,000 \quad F = ? \quad i = 10\% \text{ per year} \]

\[ F = P(1+i)^n = 1,000(1.1)^6 = \$1,771.56 \]

Solution, Future Balance, \( F = P(1+i)^n = 1,000(1.1)^6 = \$1,771.56 \)

Example 2-2  Single Payment Present-Worth Factor

Calculate the present value of a $1,000 payment to be received six years from now if interest is 10% per year compounded annually.

\[ P = \frac{F}{(1+i)^n} = \frac{\$1,000}{(1.1)^6} = \$564.50 \]

Solution, Present Value, \( P = \frac{F}{(1+i)^n} = \frac{\$1,000}{(1.1)^6} = \$564.50 \)

Single Payment Present-Worth Factor

By simply re-arranging text Equation 2-1 we can solve for the present value \( P \) given a future value, \( F \) as follows:

\[ P = F \left( \frac{1}{1+i} \right)^n \]

The equation \( 1/(1+i)^n \) is called the "single payment present worth factor," and is designated by the symbol, \( P/F, i, n \)
Example 2-2  Single Payment Present-Worth Factor

Calculate the present value of a $1,000 payment to be received six years from now if interest is 10% per year compounded annually.

\[ P = ? \]

\[ 0 \] \hspace{1cm} \[ F = $1,000 \] \hspace{1cm} \[ ^6 \]

\[ P = \frac{F}{(1+i)^n} = \frac{$1,000}{(1.1)^6} = $564.50 \] or,

\[ P = $1,000\left(\frac{1}{(1+i)^n}\right) = ? \]

\[ 0 \] \hspace{1cm} \[ F = $1,000 \] \hspace{1cm} \[ ^6 \]

Pg 19

Summary of Compound Interest Formulas

Single Payment Compound-Amount Factor

\[ F = (1+i)^n \]

Single Payment Present-Worth Factor

\[ P = \frac{F}{(1+i)^n} \]

Uniform Series Compound-Amount Factor

\[ A = \frac{F}{(1+i)^n - 1} \]
Summary of Compound Interest Formulas

Sinking-Fund Deposit Factor
\[ A = \frac{F}{A/F_{i,n}} \]

Capital-Recovery Factor
\[ A = \frac{P}{A/P_{i,n}} \]

Uniform Series Present-Worth Factor
\[ P = A(P/A_{i,n}) \]

Example 2-7  Time Value of Money Factors and Timing Considerations

A person is to receive five payments in amounts of $300 at the end of year one, $400 at the end of each of years two, three and four, and $500 at the end of year five. If the person considers that places exist to invest money with equivalent risk at 9.0% annual interest, calculate the time zero lump sum settlement "P," and the end of year five lump sum settlement "F," that would be equivalent to receiving the end of period payments.

Example 2-7 Time Zero Lump Sum Settlement Based on End of Period Values

\[
\begin{array}{cccccc}
\text{P} & \text{=} & \text{F} & \text{=}
\
0 & \text{300} & \text{400} & \text{400} & \text{400} & \text{500}
\
1 & 0.9174 & 0.8417 & 0.7722 & 0.7084 & 0.6499
\
2 & 0.8333 & 0.7598 & 0.6930 & 0.6329 & 0.5807
\
3 & 0.7624 & 0.6944 & 0.6329 & 0.5807 & 0.5353
\
4 & 0.7013 & 0.6410 & 0.5902 & 0.5454 & 0.5049
\
5 & 0.6499 & 0.5962 & 0.5484 & 0.5049 & 0.4656
\end{array}
\]

or,

\[
\begin{array}{cccccc}
\text{P} & \text{=} & \text{F} & \text{=}
\
0 & \text{300} & \text{400} & \text{400} & \text{400} & \text{500}
\
1 & 2.5313 & 0.9174 & 0.6499
\
2 & 1.9863 & 2.5313 & 0.6499 & 0.5353
\
3 & 1.6667 & 1.9863 & 0.5353 & 0.4656
\
4 & 1.4641 & 1.6667 & 0.4656 & 0.4068
\
5 & 1.3001 & 1.4641 & 0.4068 & 0.3584
\end{array}
\]

Example 2-7 Time Value of Money Factors and Timing Considerations

Next, determine the five equal end of year payments "A," at years one through five that would be equivalent to the stated payments.

Finally, recalculate the present value assuming the same annual payments are treated first, as beginning of period values and second, as mid-period values.
Example 2-7 Closer Look at the P/A Factor

\[ P = ? \]

\[
\begin{array}{cccccc}
\text{F} & \$400 & \$400 & \$400 & \$400 & \$400 \\
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\[ 2.5313 \]

\[ 400(P/A_{9\%,3}) = $1,012.52 \] at beginning of year 2, or end of year 1. This is still a future value at year 1, not the desired sum at 0, so

\[ P = 1,012.52(P/F_{9\%,1}) = $928.92 \]

or,

\[ P = 400(P/A_{9\%,3})(P/F_{9\%,1}) \]

Example 2-7 Expansion of FV Calculation

\[ F = ? \]

\[
\begin{array}{cccccc}
\text{F} & \$300 & \$400 & \$400 & \$400 & \$500 \\
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\[ 1.4116 \]

\[ 1.2950 \]

\[ 1.1881 \]

\[ 1.0900 \]

\[ F = 300(F/P_{9\%,4})+400(F/P_{9\%,3})+400(F/P_{9\%,2})+400(F/P_{9\%,1})+500 = $2,353 \]

or,

\[ F = 1,528(F/P_{9\%,5}) = $2,353 \]

Example 2-7 FV at End of Year 5 Value

\[ F = 300(F/P_{9\%,4})+400(F/A_{9\%,3})(F/P_{9\%,1})+500 = $2,353 \]

or,

\[ 3.2781 \]

\[ F = 400(F/A_{9\%,3}) = $1,311 , but it is not a year 5 future value! This is still a present sum relative to the desired future value, so; \]
Example 2-7 Expansion of FV Calculation

\[ F = 400(F/A_{9\%,3})(F/P_{9\%,1}) = \$1,429 \]
\[ F = 400 \times 3.2781 \times 1.0900 = \$1,429 \]

Example 2-7 FV at End of Year 5 Value

\[ F = 300(F/P_{9\%,4})+400(F/P_{9\%,3})+400(F/P_{9\%,2})+400(F/P_{9\%,1})+500 = \$2,353 \]
\[ F = 300 \times 1.4116 + 400 \times 1.2950 + 400 \times 1.1881 + 400 \times 1.0900 + 500 = \$2,353 \]

Example 2-7 Equivalent Annual Cost

\[ A = 1,529(A/P_{9\%,5}) = \$393 \]
\[ A = 2,353(A/P_{9\%,5}) = \$393 \]

2.8 Arithmetic Gradient Series

\[ A = B \pm g(A/G_{i,n}) \]

Where \( A/G_{i,n} = (1/i) \times [n/(1+i)^n-1] \)

"n" includes the base year as the mathematical development is based on applying the gradient \( n-1 \) times.

\[ A/G \text{ Factor developed in Appendix E, pg 788} \]
Rule of 72
Number of periods to double your money:
\[ \frac{72}{\text{Compound interest rate} \times 100} \]
Interest required to double your money:
\[ \frac{72}{\text{Number of years}} \]

Rule of 114
Number of periods to triple your money:
\[ \frac{114}{\text{Compound interest rate} \times 100} \]
Interest required to triple your money:
\[ \frac{114}{\text{Number of years}} \]

Continuous Interest on Discrete Values
(not covered in EBGN/CHEN 321)
\[
\begin{align*}
F/P_{r,n} &= e^{r n} \\
P/F_{r,n} &= 1/e^{r n} \\
F/A_{r,n} &= (e^{r n} - 1)/(e^r - 1) \\
A/F_{r,n} &= (e^r - 1)/(e^{r n} - 1) \\
P/A &= (e^{r n} - 1)/(e^r - 1)e^{r n} \\
A/P_{r,n} &= (e^r - 1)e^{r n}/(e^{r n} - 1)
\end{align*}
\]
\[ r = \text{nominal interest rate compounded continuously} \]
\[ n = \text{number of discrete evaluation periods} \]
\[ e = \text{base of natural log (ln)} = 2.7183 \ldots \]

Overview of Continuous Interest
- Same timing assumptions as discrete compounding
- You can calculate the effective rate from a continuous rate using the formula:
  \[ E = e^r - 1 \]
- The Effective rate determined on a daily basis will not be significantly different than a continuous interest rate.
2.3 Nominal, Period and Effective Interest

Nominal = Annual

\[
\text{Period Interest Rate}, i = \frac{\text{Nominal Interest Rate}}{\# \text{ Compounding Periods Per Year}, m}
\]

Effective Interest Rate, \( E \) = Annual Percentage Yield, APY

\[
E = (1+i)^m - 1 \quad \text{(Textbook Eq. 2-9)}
\]

Effective Interest Rates (or APY’s) generate annual interest equivalent to a nominal rate compounded "m" times throughout the year.

Re-arranging Eq. 2-9; the equivalent period interest rate \( i \), required to achieve a desired Effective rate \( E \), \( i = (1+E)^{1/m} - 1 \)

---

**Explanation of Effective Interest Rate, \( E \)**

\[
P \quad 0 \quad 1 \quad 2 \ldots \ldots \quad m \text{ periods/year}
\]

\[
F_1 = P(F/P, i, m) = P(1+i)^m
\]

\[
P \quad 0 \quad 1 \text{ period/year}
\]

\[
F_2 = P(F/P, E, 1) = P(1+E)^1
\]

Since the initial principal, "\( P \)" is the same in each case, set \( F_1 = F_2 \) to make the total annual interest the same for both cases as follows:

\[
\text{Effective Annual Interest, } E = (1+i)^m - 1 = \text{APY}
\]

---

**Nominal, Period and Effective Interest**

Nominal Rate = 5.0% or 0.05, compounded daily.

Daily Period Interest Rate = \( \frac{0.05}{365} = 0.000137 \) or 0.0137%

Effective Rate, \( E \) = \( (1+0.000137)^{365} - 1 = 5.127\% = \text{APY} \)

Note: Continuous compounding on discrete sums will not be significantly different than an effective annual interest rate determined on a daily basis.

---

**Variation on Period and Effective Interest**

Assume a company wanted a 10% annual rate of return but was working through a cash flow model based on monthly values:

Monthly Period Interest Rate = \( \frac{0.10}{12} = 0.008333 \) or 0.8333% \textbf{Incorrect} \n
Resulting Effective Rate, \( E \) = \( (1+0.008333)^{12} - 1 = 0.10471 \text{ or } 10.471\% \)

The correct period interest to effectively yield 10% (E=10%) per year is determined from Equation 2-9, re-arranged to solve for \( i \) as follows;

\[
i = (1+E)^{1/m} - 1 = (1.1)^{1/12} - 1 = 0.087974 \text{ or } 0.7974\% \text{ per month}
\]
Chapter 3

- Income Producing Criteria
  - Rate of Return
  - Growth Rate of Return
  - Net Present Value
  - Benefit/Cost and Present Value Ratios
- Service Evaluations
  - Incremental Analysis
  - Present, Annual, or Future Cost Analysis

Example 3-1 Present Worth Revenue Equals Break-even Acquisition Cost

Determine the present worth of the revenue streams "I", given in alternatives "A" and "B" for minimum rates of return of 10% and 20%. This gives the initial cost that can be incurred to break-even with the 10% or 20% rate of return. Note that the cumulative revenues are the same for the “A” and “B” alternatives but the timing of the revenues is very different.

Example 3-1 Solution: Case A

\[
P_A = \[200 + 100(A/G_{10\%,4})\]P/A_{10\%,4} = $1,072
\]

\[
P_A = \[200 + 100(A/G_{20\%,4})\]P/A_{20\%,4} = $848
\]
Example 3-1 Solution Case B

B) P=? I=500 I=400 I=300 I=200

0 1 2 3 4

B) i = 10%, P

B = [500 – 100(A/G 10%,4)](P/A 10%,4)

= $1,147

i = 20%, P

B = [500 – 100(A/G 20%,4)](P/A 20%,4)

$965

Example 3-3 Rate of Return (ROR)

If you pay $20,000 for the asset in Example 3-2, what annual compound interest rate of return on investment dollars will be received?

C=20,000 I=2,000 I=2,000 ... I=2,000

0 1 2 . . . . . . . 10 L=25,000

The only unknown in this problem is the rate of return, "i". A present, future or annual worth equation may be used to obtain "i" by trial and error calculation.

Example 3-3 Solution PW Equation

C=20,000 I=2,000 I=2,000 ... I=2,000

0 1 2 . . . . . . . 10 L=25,000

Present Worth (PW) Equation at Time 0 to Determine "i"

20,000 = 2,000(P/A, i,10) + 25,000(P/F, i,10)

Mathematically the equation is:

20,000 = 2,000[(1 + i)^10 - 1] / [(1 + i)^10] + 25,000[1 / (1 + i)^10]

Arithmetic Average Income = 2,000

Cumulative Initial Costs = 20,000

i = 10% = 2,000(6.145) + 25,000(.3855) = 21,930

i = ?

i = 12% = 2,000(5.650) + 25,000(.3220) = 19,350
Example 3-3 Solution by Interpolation

Because there are no 11% tables in Appendix A, interpolate between the 10% and 12% values:

\[ i = 10\% + \frac{2\%}{(21,930 - 20,000)/(21,930 - 19,350)} = 11.5\% \]

This answer can also be determined graphically.

---

Example 3-3 Solution (Continued…)

On the previous diagram, two triangles were formed. The small triangle with sides "a" and "c" is geometrically similar to the larger triangle with sides "b" and "d" since both triangles have equal angles. Therefore, the sides of the two triangles are proportional.

\[ \frac{a}{b} = \frac{c}{d}, \text{ therefore } a = b \cdot \frac{c}{d} \text{ and } b, c \text{ and } d \text{ are known} \]

Substituting these values gives:

\[ a = (12\% - 10\%)(21,930 - 20,000)/(21,930 - 19,350) = 1.5\% \]

Rate of Return, \( i = 10\% + 1.5\% = 11.5\% \)
Example 2-17 A/P, Factor Illustration

What annual end of year mortgage payments are required to pay off a $10,000 loan in five years if interest is 10% per year?

\[
P = 10,000, \quad A = ?
\]

\[
A = 10,000 \left( \frac{1}{A/P, 10\%, 5} \right) = 2,638 \text{ per year}
\]

Solution:

\[
A = 10,000 \left( \frac{1}{A/P, 10\%, 5} \right) = 2,638 \text{ per year}
\]

Example 2-17 – Loan Amortization

<table>
<thead>
<tr>
<th>Yr</th>
<th>Beg. Balance</th>
<th>Payment</th>
<th>Interest</th>
<th>Principal</th>
<th>Ending Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>2,638</td>
<td>1,000</td>
<td>1,638</td>
<td>8,362</td>
</tr>
<tr>
<td>2</td>
<td>8,362</td>
<td>2,638</td>
<td>836</td>
<td>1,802</td>
<td>6,560</td>
</tr>
<tr>
<td>3</td>
<td>6,560</td>
<td>2,638</td>
<td>656</td>
<td>1,982</td>
<td>4,578</td>
</tr>
<tr>
<td>4</td>
<td>4,578</td>
<td>2,638</td>
<td>458</td>
<td>2,180</td>
<td>2,398</td>
</tr>
<tr>
<td>5</td>
<td>2,398</td>
<td>2,638</td>
<td>240</td>
<td>2,398</td>
<td>0</td>
</tr>
</tbody>
</table>

Factors to Remember in Bond Evaluations

1. at maturity the holder will receive its "face value" as salvage or terminal value
2. bond cost or value will vary as market interest rates fluctuate up and down in general money markets
3. bond call privileges are written into most corporate or municipal bond offerings
Bond Evaluation

• The value of a bond is the present worth of all future cash flows at the market interest rate.

• A bond's rate of return is the I value that makes the PW revenue equal the PW cost.

Example 3-11 New Bond Rate of Return

Calculate the bond rate of return for a new issue of $1,000 bonds with maturity date twenty years after the issuing date, if the new bond pays interest of $40 every six month period.

Example 3-11 Solution

- $1,000  $40  $40  $40  $1,000
  0  1  2 . . . . . . . . . . . 40 semi-annual

PW Eq:  0 = -1,000 + 40(P/A, i, 40) + 1,000(P/F, i, 40)

Since initial investment and maturity value are the same:

ROR, i = 40/1,000 = 4.0% per semi-annual period

The nominal ROR is 4.0% x 2 or 8.0%, which bond brokers often refer to as the bond "Yield to Maturity," for which the acronym "YTM" is utilized.

6 years Later (28 semi annual periods remaining)

• Market interest rates have moved up, assume the Bond described in the prior example now sells for $800. Calculate the Bonds Yield to Maturity, Coupon Yield, and Current Yield.
Example 3-12 Solution

\[ \begin{array}{cccccc}
0 & \$800 & \$40 & \$40 & \ldots & \$1,000 \\
1 & & & & & \$40 \\
2 & & & & & \$40 \\
28 & & & & & \\
\end{array} \]

PW Eq: \[ 800 = 40(\text{P/A}_{i,28}) + 1,000(\text{P/F}_{i,28}) \]

\[ i = 6\% = 40(13.4062) + 1,000(0.1956) = 731.85 \]

\[ i = 5\% = 40(14.8981) + 1,000(0.2551) = 851.02 \]

By interpolation \( i = 5.43\% \) per semi-annual period.

Yield to Maturity = Nominal ROR = 5.43\% x 2 = 10.86\%

Current Yield = Annual Interest / Cost = 80/800 = 10.0\%

Coupon Yield = Annual Interest / Par Value = 8.0\%

Definition of \( i^* \)

A compound interest measure of opportunity foregone if a different investment alternative is selected.

Discount Rate, \( i^* \)

- Minimum Acceptable Rate of Return
- Opportunity Cost of Capital
- Financial Cost of Capital
- Weighted Average Cost of Capital
- Weighted Average Financial Cost of Capital
- Cost of Capital
- Hurdle Rate

3.10 Net Present Value (NPV)

\[ (\text{NPV}) = \text{Present Worth Revenues or Savings @} \ i^* - \text{Present Worth Costs @} \ i^* \]

or, \[ = \text{Present Worth Positive Cash Flows and Negative Cash Flows Discounted @} \ i^* \]

NPV > 0 indicates a satisfactory investment
NPV = 0 is an economic breakeven
NPV < 0 is economically unsatisfactory.
Example 3-21 ROR, & NPV

A five-year project requires investments of $120,000 at time zero and $70,000 at the end of year one to generate revenues of $100,000 at the end of each of years two through five. The investor’s minimum rate of return is 15.0%. Calculate the Project ROR. Also, calculate the NPV. Calculate the project payback period and finally, draw an NPV Profile to show how the value of the project is impacted by the selected discount rate.

Example 3-21 Solution

Rate of Return (ROR):

\[
PW \text{ Eq: } 0 = -120,000 - 70,000(P/F_{i=1}) + 100,000(P/A_{i=4})(P/F_{i=1})
\]

@ 25% = 12,928

@ 30% = -7,212

\[i = 25\% + 5\%(12,928 / 20,140) = 28.2\% > 15\%, \text{ acceptable}\]

Net Present Value (NPV) @ \(i^* = 15\%\)

\[
-120,000 - 70,000(P/F_{15\%,1}) + 100,000(P/A_{15\%,4})(P/F_{15\%,1})
\]

= $67,389 > 0, acceptable

Payback

\[
-120,000 \quad -190,000 \quad -90,000 \quad 10,000 \quad 90,000 \quad 190,000
\]

0 1 2 3 4 5

2 Years + ( 1 Year \times \frac{90,000}{100,000} ) = 2.9 Yrs

Payback is also a measure of financial risk expressed in time, it is not an overall economic measure of value added from the investments. Payback neglects time value of money.
NPV Profile
(A graphical illustration of Net Present Value vs. i*)

Project Rate of Return, 28.1%

Benefit-Cost Ratio, (B/C Ratio)

\[ \text{B/C Ratio} = \frac{\text{PV Positive Cash Flow @ } i^*}{\text{PV Negative Cash Flow @ } i^*} \]

B/C Ratio > 1.0 indicates satisfactory economics
B/C Ratio = 1.0 indicates break-even economics
B/C Ratio < 1.0 indicates unsatisfactory project economics

Text Problem 3-20  Pg. 167-68

Problem 3-20, 21 or 22 Solutions

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>14,000</td>
<td>8,000</td>
<td>6,000</td>
<td>4,400</td>
<td>2,800</td>
<td></td>
</tr>
<tr>
<td>-Royalty Cost</td>
<td>-1,750</td>
<td>-1,000</td>
<td>-750</td>
<td>-550</td>
<td>-350</td>
<td></td>
</tr>
<tr>
<td>Net Revenue</td>
<td>12,250</td>
<td>7,000</td>
<td>5,250</td>
<td>3,850</td>
<td>2,450</td>
<td></td>
</tr>
<tr>
<td>-Operating Cost</td>
<td>-1,750</td>
<td>-1,000</td>
<td>-750</td>
<td>-500</td>
<td>-250</td>
<td></td>
</tr>
<tr>
<td>-Mine Develop.</td>
<td>-7,500</td>
<td>-2,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Equipment</td>
<td>-6,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Lease Bonus</td>
<td>-1,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before-Tax CF</td>
<td>-8,500</td>
<td>1,300</td>
<td>6,000</td>
<td>4,500</td>
<td>3,350</td>
<td>2,200</td>
</tr>
</tbody>
</table>

Sol. Man. Pg 61-3
Problem 3-20, 21 or 22 Solutions

\[ \text{NPV} @ 15\% = -8,500 + 1,300(P/F_{15,1}) + 6,000(P/F_{15,2}) + 4,500(P/F_{15,3}) \\
+ 3,350(P/F_{15,4}) + 2,200(P/F_{15,5}) = +$3,135 > 0, \text{ accept} \]

\[ \text{NPV} @ 25\% = +$777 \]

\[ \text{NPV} @ 30\% = -$136, i = 25\% + 5\%(777/(777+136)) = 29.3\% > i^* = 15\% \]

By financial calculator, \( i = ROR = 29.2\% > i^* = 15\% \), accept

Problem 3-20 Breakeven Solution

\[ X = \text{Break-even Uniform Selling Price Per Unit:} \]

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>175X</td>
<td>100X</td>
<td>75X</td>
<td>55X</td>
<td>35X</td>
<td></td>
</tr>
<tr>
<td>-Royalty Cost</td>
<td>-21.9X</td>
<td>-12.5X</td>
<td>-9.4X</td>
<td>-6.9X</td>
<td>-4.4X</td>
<td></td>
</tr>
<tr>
<td>Net Revenue</td>
<td>153.1X</td>
<td>87.5X</td>
<td>65.6X</td>
<td>48.1X</td>
<td>30.6X</td>
<td></td>
</tr>
<tr>
<td>-Operating Cost</td>
<td>-1,750</td>
<td>-1,000</td>
<td>-750</td>
<td>-500</td>
<td>-250</td>
<td></td>
</tr>
<tr>
<td>-Mine Develop</td>
<td>-7,500</td>
<td>-2,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Mine Equip.</td>
<td>-6,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Lease Bonus</td>
<td>-1,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before-Tax CF</td>
<td>-8,500</td>
<td>153.1X</td>
<td>87.5X</td>
<td>65.6X</td>
<td>48.1X</td>
<td>30.6X</td>
</tr>
</tbody>
</table>

\[ \text{PW Eq:} 0 = -8,500 + (153.1X - 10,950)(P/F_{15,1}) + (87.5X - 1,000)(P/F_{15,2}) \\
+ 65.6X - 750)(P/F_{15,3}) + (48.1X - 500)(P/F_{15,4}) \\
+ (30.6X - 250)(P/F_{15,5}) \]

\[ 0.8696 \quad 0.7561 \quad 0.6575 \quad 0.5718 \quad 0.4972 \]

\[ 0 = -19,681 + 285.1X \]

Sol. Man. Pg 61-3
**Problem 3-20, 21 or 22 Breakeven Solution**

PW Eq: \[ 0 = -8,500 + (153.1X - 10,950)(P/F_{15,1}) + (87.5X - 1,000)(P/F_{15,2}) + (65.6X - 750)(P/F_{15,3}) + (48.1X - 500)(P/F_{15,4}) + (30.6X - 250)(P/F_{15,5}) \]

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P/F_{15,1} )</td>
<td>0.8696</td>
<td>(P/F_{15,2} )</td>
<td>0.7561</td>
</tr>
<tr>
<td>(P/F_{15,3} )</td>
<td>0.6575</td>
<td>(P/F_{15,4} )</td>
<td>0.5718</td>
</tr>
<tr>
<td>(P/F_{15,5} )</td>
<td>0.4972</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 0 = -19,681 + 285.1X \]

Present Worth Net Cost

Sol. Man. Pg 61-3

**Problem 3-20, 21 or 22 Breakeven Solution**

PW Eq: \[ 0 = -8,500 + (153.1X - 10,950)(P/F_{15,1}) + (87.5X - 1,000)(P/F_{15,2}) + (65.6X - 750)(P/F_{15,3}) + (48.1X - 500)(P/F_{15,4}) + (30.6X - 250)(P/F_{15,5}) \]

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P/F_{15,1} )</td>
<td>0.8696</td>
<td>(P/F_{15,2} )</td>
<td>0.7561</td>
</tr>
<tr>
<td>(P/F_{15,3} )</td>
<td>0.6575</td>
<td>(P/F_{15,4} )</td>
<td>0.5718</td>
</tr>
<tr>
<td>(P/F_{15,5} )</td>
<td>0.4972</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 0 = -19,681 + 285.1X \]

Present Worth Net Production x Selling Price, \(X = PW\) Net Revenue

Sol. Man. Pg 62

**Problem 3-20, 21 or 22 Breakeven Solution**

PW Eq: \[ 0 = -8,500 + (153.1X - 10,950)(P/F_{15,1}) + (87.5X - 1,000)(P/F_{15,2}) + (65.6X - 750)(P/F_{15,3}) + (48.1X - 500)(P/F_{15,4}) + (30.6X - 250)(P/F_{15,5}) \]

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P/F_{15,1} )</td>
<td>0.8696</td>
<td>(P/F_{15,2} )</td>
<td>0.7561</td>
</tr>
<tr>
<td>(P/F_{15,3} )</td>
<td>0.6575</td>
<td>(P/F_{15,4} )</td>
<td>0.5718</td>
</tr>
<tr>
<td>(P/F_{15,5} )</td>
<td>0.4972</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 0 = -19,681 + 285.1X \]

\[ 19,681 = 285.1X \]

Or more generically, PW Cost = PW Revenue

Sol. Man. Pg 62
### Problem 3-20, 21 or 22 Breakeven Solution

**PW Eq:**

\[
0 = -8,500 + (153.1X - 10,950)(P/F_{15,1}) + (87.5X - 1,000)(P/F_{15,2}) + (65.6X - 750)(P/F_{15,3}) + (48.1X - 500)(P/F_{15,4}) + (30.6X - 250)(P/F_{15,5})
\]

Using the values:

\[
\begin{align*}
P/F_{15,1} &= 0.8696 \\
P/F_{15,2} &= 0.7561 \\
P/F_{15,3} &= 0.6575 \\
P/F_{15,4} &= 0.5718 \\
P/F_{15,5} &= 0.4972
\end{align*}
\]

\[
0 = -19,681 + 285.1X
\]

\[
19,681 = 285.1X
\]

\[
X = \frac{19,681}{285.1} = 69.03 \text{ per unit}
\]

---

### 3.14 ROR, NPV and PVR Analysis

**For Service Producing Investments With Equal Lives**

For rate of return, net value or ratio analysis of alternatives that provide a service, investors must make an "incremental analysis" of alternatives. Incremental analyses are made to determine if the additional up front investment(s) in the more capital-intensive alternative generates sufficient reductions in downstream operating costs (incremental savings) to justify the investment.

---

### Example 3-27 Cash Flow Solution

**Incremental Setup Approach #2 – Cash Flow Sign Convention**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th>B</th>
<th></th>
<th></th>
<th>A-B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-200</td>
<td>-220</td>
<td>-240</td>
<td>-260</td>
<td>-290</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

**Using either approach, solving for the incremental rate of return using trial and error provides the following:**

\[
\begin{align*}
@ 30\% &= 16 \\
@ 40\% &= -19
\end{align*}
\]

Interpolating: \( ROR, i = 30\% + \frac{10\%}{(16 / (16 + 19))} = 34.6\% \)

\( 34.6\% > 20\%, \text{ economics of automated equipment acceptable} \)
Example 3-27 Cash Flow Solution

Incremental A-B Net Present Value @ 20%:

\[ 0 = -200 \times (P/F,20,1) + 90 \times (P/F,20,2) + 100 \times (P/F,20,3) + 160 \times (P/F,20,4) \]
\[ = +64.2 > 0, \text{ so accept automated equipment} \]

Incremental A-B Present Value Ratio:

\[ 64.2 / 200 = 0.32 > 0, \text{ so accept automated equipment.} \]

3.15 Cost Analysis of Services Producing Alternatives That Provide the Same Service Over the Same Period of Time

- It is equally valid to analyze the present, annual or future cost of providing a service for a common evaluation life.
- The minimum cost analysis may be based on using the cost and revenue sign convention where costs are positive and revenues are negative. Note this is the opposite of the cash flow sign convention where revenues are positive and costs negative. Consistency in application and proper interpretation of results is really the key issue as either method is valid.

Example 3-28 Solution

The incremental analysis presented in Example 3-27 was based on looking at the difference in the A – B costs or cash flows. If you consider the difference in the

\[ \text{PWC}_A - \text{PWC}_B \text{ or, } -816.2 - (-880.4) = +64.2 \]

This was the incremental NPV for A–B.
Chapter Four

- Mutually Exclusive Alternatives
  - Examples include Develop vs Sell or, Joint Ventures, Buy vs. Explore, Financial Constraints, or Manpower Constraints.
  - When applying criterion, biggest economic measure not always best!
  - Incremental analysis is the key concept!
- Non-Mutually Exclusive Alternatives
  - Ranking Exploration Prospects
  - More than one alternative may be selected
  - Objective to maximize cumulative wealth!

Chapter Four

• Mutually Exclusive Alternatives
  - Examples include Develop vs Sell or, Joint Ventures, Buy vs. Explore, Financial Constraints, or Manpower Constraints.
  - When applying criterion, biggest economic measure not always best!
  - Incremental analysis is the key concept!
• Non-Mutually Exclusive Alternatives
  - Ranking Exploration Prospects
  - More than one alternative may be selected
  - Objective to maximize cumulative wealth!

Inflation

- Inflation is defined as a persistent rise in the prices of a "Consumer Price Index" type basket of goods, services and commodities that is not offset by increased productivity.
  - The Federal Reserve might define inflation as the result of too many dollars chasing too few goods.
- Core measure based on the personal consumption expenditure "PCE" Index
- Deflation refers to an overall decline in the prices for a similar basket of goods and services.

Equivalent Escalated Dollar and Constant Dollar Present Value Calculations

Today’s $ Escalate Using P/F, e Escalated Dollars Discount Using P/F, i* Net Present Value

\[ \begin{align*}
  e &= \text{escalation rate} \\
  f &= \text{inflation rate} \\
  i^* &= \text{escalated } S\text{ discount rate} \\
  i^* &= \text{constant $ discount rate} \\
  i &= \text{escalated } S\text{ rate of return} \\
  i' &= \text{constant $ rate of return}
\end{align*} \]

Net Present Value

\[ \text{Discount Using } P/F, i^* \]

Constant Dollars

\[ \text{Discount Using } P/F, i' \]

Section 5.3 Summary, pg. 306

Variables related to escalated and constant dollar calculations:

\[ \begin{align*}
  e &= \text{parameter escalation rate(s)} \\
  f &= \text{annual inflation rate} \\
  i^* &= \text{escalated } S\text{ discount rate} \\
  i^* &= \text{constant } S\text{ discount rate} \\
  i &= \text{escalated } S\text{ rate of return or period interest rate} \\
  i' &= \text{constant } S\text{ rate of return or period interest rate}
\end{align*} \]

Eq 5.1: \( (1+i) = (1+f)(1+i') \)

Or, rearranged: \( i' = \frac{(1+i)}{(1+f)} - 1 \)

Common approximation: \( i' = i - f \)
6.5 Expected Value Analysis

- **Expected value** is defined as the difference between expected profits and expected costs.
- **Expected profit** is the probability of receiving a certain profit times the profit.
- **Expected cost** is the probability that a certain cost will be incurred times the cost.
- A positive expected value is necessary, but not always a sufficient condition for an economically satisfactory investment in light of the perceived uncertainty and financial risk.

Example 6-4 Expected Value Analysis of a Gambling Game

A wheel of fortune in a gambling casino has 54 different slots in which the wheel pointer can stop. 4 of the 54 slots contain the number 9. For $1 bet on hitting a 9, the gambler wins $10 plus the return of the $1 bet if he or she succeeds.

What is the expected value of this gambling game?

What is the meaning of the expected value result?

Example 6-4 Solution

Probability of Success = 4/54
Probability of Failure = 50/54
Expected Value = Expected Profit – Expected Cost
               = (4/54)($10) – (50/54)($1) = - $0.185

Straight Line Depreciation (Financial)
Not Covered in EBGN/CHEN321

(Cost Basis – Salvage Value) / Depreciation Life = Yearly Depreciation
Table 7-3 MACRS Depreciation Rates

<table>
<thead>
<tr>
<th>Year</th>
<th>3-Year</th>
<th>5-Year</th>
<th>7-Year</th>
<th>10-Year</th>
<th>15-Year</th>
<th>20-Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.3333</td>
<td>.2000</td>
<td>.1429</td>
<td>.1000</td>
<td>.0650</td>
<td>.03750</td>
</tr>
<tr>
<td>2</td>
<td>.4445</td>
<td>.3200</td>
<td>.2449</td>
<td>.1800</td>
<td>.0950</td>
<td>.07219</td>
</tr>
<tr>
<td>3</td>
<td>.1861</td>
<td>.1920</td>
<td>.1749</td>
<td>.1440</td>
<td>.0855</td>
<td>.06677</td>
</tr>
<tr>
<td>4</td>
<td>.0861</td>
<td>.1152</td>
<td>.1249</td>
<td>.1152</td>
<td>.0770</td>
<td>.06177</td>
</tr>
<tr>
<td>5</td>
<td>.1152</td>
<td>.0893</td>
<td>.0922</td>
<td>.0693</td>
<td>.05713</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>.0576</td>
<td>.0892</td>
<td>.0737</td>
<td>.0623</td>
<td>.05285</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>.0893</td>
<td>.0655</td>
<td>.0590</td>
<td>.04522</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>.0446</td>
<td>.0655</td>
<td>.0591</td>
<td>.04462</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>.0656</td>
<td>.0655</td>
<td>.0590</td>
<td>.04461</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Etc...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 7-7
Depreciation Using Table 7-3

<table>
<thead>
<tr>
<th>Year</th>
<th>7-Yr Life Rate</th>
<th>Initial Basis</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1429</td>
<td>$100,000</td>
<td>$14,290</td>
</tr>
<tr>
<td>2</td>
<td>0.2449</td>
<td>$100,000</td>
<td>$24,490</td>
</tr>
<tr>
<td>3</td>
<td>0.1749</td>
<td>$100,000</td>
<td>$17,490</td>
</tr>
<tr>
<td>4</td>
<td>0.1249</td>
<td>$100,000</td>
<td>$12,490</td>
</tr>
<tr>
<td>5</td>
<td>0.0893</td>
<td>$100,000</td>
<td>$8,930</td>
</tr>
<tr>
<td>6</td>
<td>0.0892</td>
<td>$100,000</td>
<td>$8,920</td>
</tr>
<tr>
<td>7</td>
<td>0.0893</td>
<td>$100,000</td>
<td>$8,930</td>
</tr>
<tr>
<td>8</td>
<td>0.0446</td>
<td>$100,000</td>
<td>$4,460</td>
</tr>
</tbody>
</table>

Total: 1.0000 $100,000

Book Value

Book value is the original cost basis minus the total depreciation taken.

Note: When using straight line depreciation for the FE you can not add up the remaining depreciation to calculate the book value.

Calculate the book value of the asset in Example 7-7 after four years of MACRS depreciation?

<table>
<thead>
<tr>
<th>Year</th>
<th>7-Yr Life Rate</th>
<th>Initial Basis</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1429</td>
<td>$100,000</td>
<td>$14,290</td>
</tr>
<tr>
<td>2</td>
<td>0.2449</td>
<td>$100,000</td>
<td>$24,490</td>
</tr>
<tr>
<td>3</td>
<td>0.1749</td>
<td>$100,000</td>
<td>$17,490</td>
</tr>
<tr>
<td>4</td>
<td>0.1249</td>
<td>$100,000</td>
<td>$12,490</td>
</tr>
</tbody>
</table>

Total Depreciation: 68,760

Book Value: $100,000 – $68,760 = $31,240
The present worth of the costs for a project with an infinite life is known as a capitalized cost. It is the amount of money at time period zero needed to perpetually support the project.

Capitalized Cost = \( P = \frac{A}{i} \)

Good Luck!
If you have any questions please stop by my office and I’d be happy to answer!

Andy Pederson
Engineering Hall #125
apederso@mines.edu
apederson@mc.com
Cell: (253) 320-1485