Fluid Mechanics FE Review

I'm so sorry I had to cancel tonight's session,

Please look through these notes and review problems. I will be happy to answer any questions you have and meet with you individually if you would like. If there is enough interest, I will be happy to schedule a makeup session.

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Fluid Mechanics FE Review

These slides contain some notes, thoughts about what to study, and some practice problems. The answers to the problems are given in the last slide.

In the review session, we will be working some of these problems. Feel free to come to the session, or work the problems on your own. I am happy to answer your email questions or those that you bring to the session.

Good luck!
Fluid Mechanics FE Review

MAJOR TOPICS
Fluid Properties
Fluid Statics
Fluid Dynamics
Fluid Measurements
Dimensional Analysis

**Most equations and problems taken from Professional Publications, Inc. FERC Review Course Book**

Fluid Mechanics FE Review

It will be very helpful to memorize the following concepts and equations:

- Specific weight, density, and specific gravity
- Hydrostatics pressure equation / manometry
- Force magnitude and location due to hydrostatic pressure for horizontal and vertical plane walls
- Conservation of mass / continuity
- Conservation of energy / Bernoulli and Energy Eqn
- Darcy Eqn
- Relative roughness equation
- Drag equation
- How to use the Moody Diagram
Fluid Mechanics

Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol &amp; Equation</th>
<th>Definition</th>
<th>Etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>( \rho = \frac{m}{V} )</td>
<td>mass ( \rho ) ( \frac{m}{V} )</td>
<td></td>
</tr>
<tr>
<td>Specific Weight</td>
<td>( \gamma = \rho g )</td>
<td>density ( x ) gravity</td>
<td></td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>( S_G = \frac{\rho x}{\rho_{\text{water}}\gamma_{\text{water}}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>( \mu = \frac{\tau}{\text{dy}} )</td>
<td>shear stress ( \frac{\tau}{\text{dy}} )</td>
<td></td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( \nu = \frac{\mu}{\rho} )</td>
<td>( \text{viscosity} ) ( \frac{\mu}{\rho} )</td>
<td></td>
</tr>
<tr>
<td>Ideal Gas Law</td>
<td>( p = \rho R_g T )</td>
<td>Use to find properties ( p ) ( \rho R_g T )</td>
<td>( R_g = \frac{\mu}{\text{molec. wt.}} )</td>
</tr>
</tbody>
</table>

Make sure you know the relationship between density, specific weight, and specific gravity!

Fluid Mechanics

Properties

Fluids
- Substances in either the liquid or gas phase
- Cannot support shear

Fluid Properties

- Density, \( \rho \)
- Specific Gravity, \( S_G = \frac{\rho x}{\rho_{\text{water}}\gamma_{\text{water}}} \)
- Surface Tension, \( \sigma = \frac{\text{Force}}{\text{Area}} \)
- Temperature, \( T \)
- Ideal Gas Law, \( PV = nRT \)

Make sure you use the ideal gas law when calculating properties for gasses!!
Mass & Weight

1. 10.0 L of an incompressible liquid exert a force of 20 N at the earth’s surface. What force would 2.3 L of this liquid exert on the surface of the moon? The gravitational acceleration on the surface of the moon is 1.67 m/s²

\[
F = ma \quad \Rightarrow \quad m = \frac{F}{a} = \frac{20 \text{ N}}{1.67 \text{ m/s}^2} = 12 \text{ kg}
\]

\[
\rho = \frac{m}{V} = \frac{12 \text{ kg}}{10 \text{ L}} = 1.2 \text{ kg/L}
\]

on the moon:

\[
F = ma = \rho \cdot V
\]

\[
= 0.204 \text{ kg/L} \times (2.3 \text{ L}) \times (1.67 \text{ m/s}^2) = 0.78 \text{ N}
\]

(A) 0.39 N
(B) 0.78 N
(C) 3.4 N
(D) 4.6 N

Fluid Mechanics – Fluid Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol &amp; Equation</th>
<th>Definition</th>
<th>Etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor Pressure</td>
<td>( p_v )</td>
<td>Pressure at which liquid and vapor are in equilibrium</td>
<td>Used to predict cavitation (local pressure &lt; vapor pressure)</td>
</tr>
<tr>
<td>Bulk Modulus</td>
<td>( \rho = \frac{m}{V} )</td>
<td><strong>mass</strong> / <strong>volume</strong></td>
<td></td>
</tr>
<tr>
<td>Speed of Sound</td>
<td>( c_{\text{liquid}} = \sqrt{\frac{k}{\rho}} )</td>
<td>Velocity of propagation of a small wave</td>
<td>( k = \text{Bulk modulus} ) ( k_{\text{air}} = 1.4 )</td>
</tr>
<tr>
<td></td>
<td>( c_{\text{gas}} = \sqrt{kT} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SPEED OF SOUND

\[
C = \sqrt{\frac{kR}{M}} \quad (T \text{ must be in K or } ^\circ \text{C})
\]

Mach Number

\[
M = \frac{V}{C} = \text{object velocity} / \text{speed of sound}
\]
Mach Number

2. A jet aircraft is flying at a speed of 1700 km/h. The air temperature is 20°C. The molecular weight of air is 29 g/mol. What is the Mach number of the aircraft?

\[
\text{Mach number, } M = \frac{V}{C} = \frac{1700 \text{ km/hr (1000 m/km)}}{343 \text{ m/s (343 m/s)}} = 1.38
\]

(A) 0.979  
(B) 1.38  
(C) 1.92  
(D) 5.28

Fluid Mechanics – Fluid Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol &amp; Equation</th>
<th>Definition</th>
<th>Etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Tension</td>
<td>( \sigma = \frac{F}{R} )</td>
<td>force length</td>
<td></td>
</tr>
<tr>
<td>Capillary Rise</td>
<td>( h = \frac{4\sigma \cos \beta}{\rho g d_{	ext{tube}} g} )</td>
<td>Distance a liquid will rise (or fall) in a “tube”</td>
<td></td>
</tr>
</tbody>
</table>
Surface Tension

3. A 2 mm (inside diameter) glass tube is placed in a container of mercury. An angle of 40° is measured as illustrated. The density and surface tension of mercury are 13550 kg/m³ and 37.5 x 10⁻² N/m, respectively. How high will the mercury rise or be depressed in the tube as a result of capillary action?

(A) -4.3 mm (depression)
(B) -1.6 mm (depression)
(C) 4.2 mm (rise)
(D) 6.4 mm (rise)

Fluid Mechanics

Stresses and Viscosity

Shear Stress

- Normal Component: \( \tau_N = \frac{F}{A} \)
- Tangential Component
  - For a Newtonian fluid: \( \tau_t = \mu \frac{\partial v}{\partial y} \) (\( v \) = velocity)
  - For a pseudoplastic or dilatant fluid: \( \tau_t = K \left( \frac{dv}{dy} \right)^n \)

Absolute Viscosity = \( \mu \) = Ratio of shear stress to rate of shear deformation

\( \mu < 1 \)
\( \mu > 1 \)
\( \mu = 1 \)
### Viscosity

4. A sliding-plate viscometer is used to measure the viscosity of a Newtonian fluid. A force of 25 N is required to keep the top plate moving at a constant velocity of 5 m/s. What is the viscosity of the fluid?

\[ \tau = \frac{F}{A} \]

\[ \tau = \frac{v}{\delta} \]

\[ F = \frac{\tau A}{v} \]

- (A) 0.005 N-s/m²
- (B) 0.04 N-s/m²
- (C) 0.2 N-s/m²
- (D) 5.0 N-s/m²

### Fluid Mechanics

#### Fluid Statics

**Gage and Absolute Pressure**

\[ p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atmospheric}} \]

**Hydrostatic Pressure**

\[ p = \gamma h + \rho gh \]

\[ p_2 - p_1 = -\gamma(z_2 - z_1) \]

Example (FEIM):
In which fluid is 700 kPa first achieved?

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density (kg/m³)</th>
<th>Pressure at 60 m</th>
<th>Pressure at 10 m</th>
<th>Pressure at 5 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethanol</td>
<td>789</td>
<td>7.586 kPa/m</td>
<td>8.825 kPa/m</td>
<td>9.604 kPa/m</td>
</tr>
<tr>
<td>Oil</td>
<td>800</td>
<td>8.825 kPa/m</td>
<td>10.000 kPa/m</td>
<td>11.000 kPa/m</td>
</tr>
<tr>
<td>Water</td>
<td>1000</td>
<td>9.604 kPa/m</td>
<td>11.000 kPa/m</td>
<td>12.000 kPa/m</td>
</tr>
<tr>
<td>Glycerin</td>
<td>1260</td>
<td>12.125 kPa/m</td>
<td>13.500 kPa/m</td>
<td>14.500 kPa/m</td>
</tr>
</tbody>
</table>

- (A) ethyl alcohol
- (B) oil
- (C) water
- (D) glycerin

Ans: D
**Fluid Statics**

**Pressure:**

\[ P_{\text{total}} = P_{\text{static}} + P_{\text{static}} + P_{\text{atmospheric}} = P_{\text{atmospheric}} + P_{\text{vacuum}} \]

**Barometer:**

Start with \( P_0 \) on one end, add or subtract change in hydrostatic pressure to \( P_0 \) at other end (**TAKE A JOURNEY through**)

\[ P_0 - P_a = \gamma_2 h_2 - \gamma_1 h_1 \]

---

**Fluid Statics**

**Barometer**

\[ P_a - P_0 = \rho gh \]  

[SI]

---

Atmospheric Pressure

*Remember - this value affects the pressure. Don't neglect it!*

\[ P_a - P_0 = \rho gh \]
Manometry

5. An open water manometer is used to measure the pressure in a tank. The tank is half-filled with 50,000 kg of a liquid chemical that is not miscible in water. The manometer tube is filled with liquid chemical. What is the pressure in the tank relative to the atmospheric pressure?

(A) 1.4 kPa
(B) 1.9 kPa
(C) 2.4 kPa
(D) 3.4 kPa
Fluid Statics

 Fluid Statics

\[ F = \rho g \frac{1}{2} A \sin \theta \]

\[ \text{Volume of pressure prism} \quad V = \frac{J_{zz}}{\rho g A} \]

\[ \text{Buoyancy} \quad F_{\text{buoy}} = \frac{V}{\text{fluid}} \text{displaced} \]

Hydrostatics

6. A 6m x 6m x 6m vented cubical tank is half-filled with water; the remaining space is filled with oil (SG=0.8). What is the total force on one side of the tank?

(A) 690 kN
(B) 900 kN
(C) 950 kN
(D) 1.0 MN

\[ P_1 = \rho_1 g \frac{1}{2} h (\text{water}) = 2.354 \times 10^5 \text{ N} \]

\[ P_2 = P_1 + \rho_2 g \frac{1}{2} h (\text{water}) = 5.297 \times 10^5 \text{ N} \]

\[ F = \frac{1}{2} P_1 (\frac{1}{2} h) (\text{width}) + \frac{1}{2} P_2 (\frac{1}{2} h) (\text{width}) + \rho_2 (\frac{1}{2} h) (\text{width}) = 3.01 \times 10^7 \text{ N} \]
Fluid Statics Examples

Example 1 (FEIM): The tank shown is filled with water. What is the force that acts on a 1 m width of the inclined portion?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

Forces on Submerged Surfaces

\[ R = pA \]
\[ \bar{p} = \frac{1}{2} \rho g (h_1 + h_2) \]

The average pressure on the inclined section is:

\[ p_{\text{ave}} = \left( \frac{1}{2} \rho g (h_1 + h_2) \right) \left( \frac{9.81 \, \text{m/s}^2}{1 \, \text{m}^2} \right) (3 \, \text{m} + 5 \, \text{m}) \]
\[ = 39122 \, \text{Pa} \]

The resultant force is

\[ R = p_{\text{ave}} A = (39122 \, \text{Pa})(2.31 \, \text{m})(1 \, \text{m}) \]
\[ = 90372 \, \text{N} \]

Fluid Statics Examples

Example 1 (FEIM): At what depth does the resultant force act?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

\[ A = bh \]
\[ I_z = \frac{b' h}{12} \]
\[ Z_c = \frac{4 \, \text{m}}{\sin 60^\circ} = 4.618 \, \text{m} \]
Fluid Mechanics

Fluid Statics

Center of Pressure

\[ y^* = \frac{\rho g I_y \sin \alpha}{p_c A} \quad [\text{SI}] \quad 23.17a \]

\[ z^* = \frac{\rho g I_\gamma \sin \alpha}{p_c A} \quad [\text{SI}] \quad 23.18a \]

If the surface is open to the atmosphere, then \( p_0 = 0 \) and

\[ p_c = \bar{p} = \rho g z_c \sin \alpha \quad [\text{SI}] \quad 23.19a \]

\[ y_{cp} - y_c = y^* = \frac{I_y}{z_c A} \quad 23.20 \]

\[ z_{cp} - z_c = z^* = \frac{I_\gamma}{z_c A} \quad 23.21 \]

Fluid Statics Examples

Example 1 (FEIM):
At what depth does the resultant force act?

The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

\[ A = bh \]

\[ I_y = \frac{b' h}{12} \]

\[ Z_s = \frac{4 \text{ m}}{\sin 60^\circ} = 4.618 \text{ m} \]

\[ Z_c = \frac{I_y}{AZ_c} = \frac{b' h}{12bhZ_s} = \frac{b'^2}{12Z_s} \]

\[ z^* = \frac{(2.31 \text{ m})^2}{(12)(4.618 \text{ m})} = 0.0963 \text{ m} \]

\[ R_{depth} = (Z_c + z^*) \sin 60^\circ = (4.618 \text{ m} + 0.0963 \text{ m}) \sin 60^\circ = 4.08 \text{ m} \]
Fluid Statics Examples

Example 2 (FEIM):
The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m³. The magnitude of the force \( F \) per meter of width to keep the gate closed is most nearly

\[
p_{\text{ave}} = \rho g z_{\text{ave}} = 1600 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times \frac{1}{2} \times 3 \text{ m} = 23544 \text{ Pa}
\]

\[
\frac{R}{w} = \frac{p_{\text{ave}} h}{(3 \text{ m})} = 70662 \text{ N/m}
\]

\[
F + F_h = R
\]

\[
R \text{ is one-third from the bottom (centroid of a triangle from the NCEES Handbook).}
\]

Taking the moments about \( R \),

\[
2F = \frac{1}{3} \left( \frac{R}{w} \right) w = \frac{70,667 \text{ N}}{3} = 23.6 \text{ kN/m}
\]

Therefore, (B) is correct.

(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m

Hydrostatics

7. A gravity dam has the cross section shown. What is the magnitude of the resultant water force (per meter of width) acting on the face of the dam?

\[
F_y = \frac{1}{2} \rho \omega L \Delta \omega
\]

\[
= \frac{1}{2} \times 1000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 30 \text{ m} \times 20 \text{ m} 
\]

\[
= 7.85 \times 10^6 \text{ N/m}
\]

\[
R_{\text{resultant}} = \sqrt{R_x^2 + R_y^2}
\]

\[
= 141.6 \times 10^6 \text{ N/m}
\]

(A) 7.85 MN/m
(B) 12.3 MN/m
(C) 14.6 MN/m
(D) 20.2 MN/m
Buoyancy

Archimedes’ Principle and Buoyancy

- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

\[ F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}} \]

8. A 35 cm diameter solid sphere (\( \rho = 4500 \text{ kg/m}^3 \)) is suspended by a cable as shown. Half of the sphere is in one fluid (\( \rho = 1200 \text{ kg/m}^3 \)) and the other half of the sphere is in another (\( \rho = 1500 \text{ kg/m}^3 \)). What is the tension in the cable?

(A) 297 N  
(B) 593 N  
(C) 694 N  
(D) 826 N
Fluid Dynamics

Continuity: Mass is conserved
\[ A_1 v_1 = A_2 v_2 \]
\[ \rho A_1 v_1 = \rho A_2 v_2 \]
For incompressible fluids, \( p \) is constant
\[ Q = A_1 v_1 = A_2 v_2 \text{ etc.} \]

General Equation:
(might be called the field eqn.)
\[ \frac{\partial p}{\partial x} + \frac{v_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + g z_2 + \frac{p_1}{\rho} + g z_1 + g \frac{V_x^2}{2} \frac{V_x^2}{2} + g \frac{V_y^2}{2} + g \frac{V_z^2}{2} \]
Exponential Fluid:
(Nonlinear fluids)
\[ \text{Exponential Fluid: } Re \leq 100 \]
Laminar Flow: \( Re < 2100 \)
Turbulent Flow: \( Re > 4000 \)

Continuity Equation Example

Example (FEIM):

The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly
(A) 1 m/s
(B) 2 m/s
(C) 4 m/s
(D) 16 m/s
\[ A_1 v_1 = A_2 v_2 \]
\[ A_1 = 4 A_2 \]
so \[ v_2 = 4 v_1 = 4 \left( \frac{4 \text{ m/s}}{1} \right) = 16 \text{ m/s} \]
Therefore, (D) is correct.
Bernoulli Equation

9. The diameter of a water pipe gradually changes from 5 cm at point A to 15 cm at point B. Point A is 5 m lower than point B. The pressure is 700 kPa at point A and 664 kPa at point B. Friction between the water and the pipe walls is negligible. What is the rate of discharge at point B?

(A) 0.0035 m³/s
(B) 0.0064 m³/s
(C) 0.010 m³/s
(D) 0.018 m³/s

Bernoulli – Flow from a Jet

10. A liquid with a specific gravity of 0.9 is stored in a pressurized, closed storage tank. The tank is cylindrical with a 10 m diameter. The absolute pressure in the tank above the liquid is 200 kPa. What is the initial velocity of a fluid jet when a 5 cm diameter orifice is opened at point A?

(A) 11.3 m/s
(B) 18.0 m/s
(C) 18.6 m/s
(D) 23.9 m/s
Flow in Pipes – Fluid Dynamics and Friction

Steady, Incompressible Flow

Energy equation:
\[ \frac{v^2}{2g} + z + \frac{v^2}{2g} + h_f + h_L = \text{loss due to fittings} \]

- \( h_L = C \frac{v^2}{2g} \) for fittings
- \( h_f = f \frac{v^2}{2g} \) for fittings

\[ f_L = f \left( \frac{v^2}{2g} \right) \]

Turbulent Flow

For a Newtonian fluid:
\[ D = \text{hydraulic diameter} = 4R_h \]
\[ v = \text{kinematic viscosity} \]
\[ \mu = \text{dynamic viscosity} \]

Reynolds Number

For a Newtonian fluid:
\[ Re = \frac{vD\rho}{\mu} \quad [\text{SI}] \]
\[ Re = \frac{vD}{\nu} \]

\[ D = \text{hydraulic diameter} = 4R_h \]
\[ v = \text{kinematic viscosity} \]
\[ \mu = \text{dynamic viscosity} \]

For a pseudoplastic or dilatant fluid:
\[ Re' = \frac{v^3nD^n\rho}{K \left( \frac{3n+1}{4n} \right)^{\frac{3n+1}{n}}} \]

Professional Publications, Inc.
Fluid Dynamics and Friction

11. A steel pipe with an inside diameter of 25 mm is 20 m long and carries water at a rate of 4.5 m$^3$/h. Assuming the specific roughness of the pipe is 0.00005 m, the water has an absolute viscosity of $1.00 \times 10^{-3}$ Pa-s and a density of 1000 kg/m$^3$, what is the friction factor?

(A) 0.023  
(B) 0.030  
(C) 0.026  
(D) 0.028

Fluid Mechanics

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions
- Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}$$

[U.S.] 24.30b

$$h_{L,\text{fitting}} = C \left( \frac{v^2}{2g} \right)$$ 24.31

Entrance and Exit Losses
- When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:

<table>
<thead>
<tr>
<th>Type</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>sharp exit</td>
<td>$C = 1.0$</td>
</tr>
<tr>
<td>protruding pipe exit</td>
<td>$C = 0.8$</td>
</tr>
<tr>
<td>sharp entrance</td>
<td>$C = 0.5$</td>
</tr>
<tr>
<td>rounded entrance</td>
<td>$C = 0.1$</td>
</tr>
</tbody>
</table>
Non-Circular Conduits, Open Channel Flow, and Partially Full Pipes

Hydraulic Radius, \( R_h = \frac{\text{area in flow}}{\text{wetted perimeter}} \); Equivalent Diameter, \( D_e = 4R_h \)

Open Channel:

\[
V = \left( \frac{1}{n} \right) R_h^{2/3} S^{1/2} \quad \text{(SI units)}
\]

\[
V = \left( \frac{1.895}{n} \right) R_h^{2/3} S^{1/2} \quad \text{(U.S. units)}
\]

\( n \) = Manning's Roughness Coefficient

Partially Full Pipes:

\[
V = 0.847 \left( \frac{R_d}{4} \right)^{0.5} S^{0.5} \quad \text{(SI)}
\]

\[
V = 1.31 \left( \frac{R_d}{4} \right)^{0.5} S^{0.5} \quad \text{(U.S.)}
\]

\( c \) = Hazen-Williams Coefficient

---

Hydraulic Radius

12. What is the hydraulic radius of the trapezoidal irrigation canal shown?

(A) 1.63 m
(B) 2.00 m
(C) 2.13 m
(D) 4.00 m

\[
R_h = \frac{\text{cross-sectional area}}{\text{wetted perimeter}}
\]

\[
R_d = \frac{3m^2}{15m} \approx 0.20 m
\]
Hydraulic Grade Line and Energy Grade Line
Pump Power

Pump Power, \( P = \frac{Q^2 H_p}{2g} \)
- \( Q \): flow rate
- \( H_p \): head added by pump to the fluid

Express in watts (1.0 mm^2) or horsepower (550 ft.lb/s)

Multi-path Pipelines

2. MAIN PIPES

1) Head loss in each branch is equal: \( h_{h1} + h_{h2} \)
\[ \frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{A_1}{A_2} \quad \text{or} \quad \frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{A_1}{A_2} \]

2) Head loss between the 2 junctions is same as head loss in each branch.

3) Mass must be conserved. Therefore, the total flow rate equals the sum of the flow rate in each branch.
\[ Q_1 + Q_2 = Q_{in} \]

This simplifies to:
\[ \frac{Q_1}{Q_2} = \frac{D_1^4 V_1}{D_2^4 V_2} = \frac{D_1^4 V_1}{D_2^4 V_2} \]
Multi-path pipelines

13. The Darcy friction factor for both of the pipes shown is 0.024. The total flow rate is 300 m³/h. What is the flow rate through the 250 mm pipe?

(A) 0.04 m³/s
(B) 0.05 m³/s
(C) 0.06 m³/s
(D) 0.07 m³/s

Impluse-Momentum

Be sure to familiarize yourself with how this equation works for bends, enlargements, contractions, jet propulsion, fixed blades, moving blades, and impulse turbines.
Impulse-Momentum

\[ \sum F = Q_2 \rho_2 v_2 - Q_1 \rho_1 v_1 \quad [\text{SI}] \text{ 24.39a} \]

Pipe Bends, Enlargements, and Contractions

\[ -F_x = p_2 A_2 \cos \alpha - p_1 A_1 + Q \rho (v_2 \cos \alpha - v_1) \quad [\text{SI}] \text{ 24.39b} \]

\[ F_x = (p_2 A_2 + Q \rho v_2) \sin \alpha + \frac{m \omega_x}{g} \quad [\text{SI}] \text{ 24.40a} \]

Fluid Mechanics

Impulse-Momentum Principle

\[ \sum F = Q_2 \rho_2 v_2 - Q_1 \rho_1 v_1 \quad [\text{SI}] \text{ 24.39a} \]

Pipe Bends, Enlargements, and Contractions

\[ -F_x = p_2 A_2 \cos \alpha - p_1 A_1 + Q \rho (v_2 \cos \alpha - v_1) \quad [\text{SI}] \text{ 24.39b} \]

\[ F_x = (p_2 A_2 + Q \rho v_2) \sin \alpha + \frac{m \omega_x}{g} \quad [\text{SI}] \text{ 24.40a} \]
Fluid Mechanics 9-6b1
Impulse-Momentum Principle

Example (FEIM):
Water at 15.5°C, 275 kPa, and 997 kg/m³ enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30°. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

Ans: 13118 N

Fluid Mechanics 9-7a
Impulse-Momentum Principle

Initial Jet Velocity:  
\[ v = \sqrt{2gh} \]

Jet Propulsion:  
\[ F = \dot{m}(v_2 - v_1) \]
\[ = \dot{m}(v_2 - 0) \]
\[ = Q \rho v_2 \]
\[ = v_2 \Delta A \rho v_2 \]
\[ = A_2 \rho v_2^2 \]
\[ = A_2 \rho \left( \sqrt{2gh} \right)^2 \]
\[ = 2\gamma h A_2 \]
\[ = 2\gamma h A_3 \]
Fluid Mechanics 9-7b1
Impulse-Momentum Principle

Fixed Blades

Figure 24.9 Open Jet on a Stationary Blade

\[ F_x = -Q \rho (v_2 \cos \alpha - v_1) \]  [SI] 24.43a
\[ F_y = Q \rho v_2 \sin \alpha \]  [SI] 24.44a

Fluid Mechanics 9-7b2
Impulse-Momentum Principle

Moving Blades

Figure 24.10 Open Jet on a Moving Blade

\[ -F_x = -Q \rho (v_1 - v)(1 - \cos \alpha) \]  [SI] 24.45a
\[ F_y = Q \rho (v_1 - v) \sin \alpha \]  [SI] 24.46a
Impulse-Momentum Principle

The maximum power possible is the kinetic energy in the flow.

\[ P_{\text{max}} = \frac{Q \rho v_1^2}{2} \text{ [SI]} \]

The maximum power transferred to the turbine is the component in the direction of the flow.

\[ P_{\text{max}} = Q \rho \left( \frac{v_1}{2} \right) (1 - \cos \alpha) \text{ [SI]} \]

14. Water is flowing at 50 m/s through a 15 cm diameter pipe. The pipe makes a 90 degree bend, as shown. What is the reaction on the water in the z-direction at the bend?

(A) -44 kN
(B) -33 kN
(C) 14 kN
(D) 44 kN
Flow and Pressure Measurement

**Pitot Tube** – used to measure flow velocity

\[ v = \sqrt{\frac{2(h_0 - p_0)}{\rho}} \]

- \( h_0 \): static pressure
- \( p_0 \): stagnation pressure

**Flow Measurement**

\[ Q = \frac{C_v A_2}{\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2}} \sqrt{2g \left( h_2 - \frac{p_2}{\rho} - g z_2 \right)} \]  

**Venturi Equations**

- \( Q \): flow rate
- \( C_v \): Venturi coefficient
- \( A_2 \): throat area
- \( A_1 \): nozzle area
- \( g \): acceleration due to gravity
- \( h_2 \): distance from nozzle plane
- \( p_2 \): pressure at throat
- \( z_2 \): elevation at throat

\[ Q = C_A \sqrt{2g \left( h_2 - \frac{p_2}{\rho} - g z_2 \right)} \]  

**Orifices**

\[ Q = C_A \sqrt{2g \left( h_2 - h_1 \right)} \]

**Submerged Orifices**

\[ Q = C_A \sqrt{2g \left( h_2 - h_0 \right)} \]

---

Fluid Mechanics

Fluid Measurements

**Pitot Tube** – measures flow velocity

\[ v = \sqrt{\frac{2(p_0 - p_0)}{\rho}} \]  

- The static pressure of the fluid at the depth of the pitot tube \( (p_0) \) must be known. For incompressible fluids and compressible fluids with \( M \leq 0.3 \),

\[ v = \sqrt{\frac{2(p_0 - p_0)}{\rho}} \]  

[SI] 25.11a
Flow and Pressure Measurement

15. The density of air flowing in a duct is 1.15 kg/m$^3$. A pitot tube is placed in the duct as shown. The static pressure in the duct is measured with a wall tap and pressure gage. Use the gage readings to determine the velocity of the air.

(A) 42 m/s
(B) 102 m/s
(C) 110 m/s
(D) 150 m/s

Fluid Mechanics

Fluid Measurements

Venturi Meters – measures the flow rate in a pipe system
• The changes in pressure and elevation determine the flow rate. In this diagram, $z_1 = z_2$, so there is no change in height.

$Q = \left( \frac{C_2 A_2}{C_1 A_1} \right)^2 \sqrt{2g \left( \frac{P_1}{\gamma} + z_1 - \frac{P_2}{\gamma} - z_2 \right)}$

\[ Q \text{ in m}^3/s \]
16. Water flows out of a tank at 12.5 m/s from an orifice located 9m below the surface. The cross-sectional area of the orifice is 0.002 m², and the coefficient of discharge is 0.85. What is the diameter D, at the vena contracta?

(A) 4.2 cm  
(B) 4.5 cm  
(C) 4.7 cm  
(D) 4.8 cm
Similitude

Geometric Similarity: Model is true to length, area, volume

Kinematic Similarity: Flow regimes of model or prototype are the same

Dynamic Similarity: Ratios of all types of forces are equal for model & prototype

If the following simultaneous equations are satisfied for model & prototype:

\[
\left( \frac{\rho v^2}{g} \right)_m = \left( \frac{\rho v^2}{g} \right)_p \\
Re_m = Re_p \\
Fr_m = Fr_p \\
Ca_m = Ca_p \\
We_m = We_p
\]

In other words... Dimensionless parameters must be equal between the model and prototype
(Each dimensionless parameter is a ratio of different types of forces being exerted on the fluid)

For example:
- For completely submerged models/prototypes and pipe flow, the Reynolds numbers must be equal
- For Weirs, dams, ships, and open channels, the Froude numbers must be equal

Similitude – Dimensionless Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number</td>
<td>( Re = \frac{V \rho}{\mu} )</td>
<td>Inertial force/viscous force</td>
</tr>
<tr>
<td>Froude number</td>
<td>( Fr = \frac{V^2}{g} )</td>
<td>Inertial force/gravitational force</td>
</tr>
<tr>
<td>Mach number</td>
<td>( M = \frac{V}{c} )</td>
<td>Inertial force/compressibility force</td>
</tr>
<tr>
<td>Weber number</td>
<td>( We = \frac{V^2 \rho}{\sigma} )</td>
<td>Inertial force/surface tension force</td>
</tr>
<tr>
<td>Strouhal number</td>
<td>( St = \frac{\omega}{V} )</td>
<td>Centrifugal force/inertial force</td>
</tr>
<tr>
<td>Pressure coefficient</td>
<td>( Cp = \frac{\Delta p}{\frac{1}{2} \rho V^2} )</td>
<td>Pressure force/inertial force</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>( Cd = \frac{\text{drag}}{\frac{1}{2} \rho V^2 A} )</td>
<td>Drag force/inertial force</td>
</tr>
</tbody>
</table>
Similitude

17. A nuclear submarine is capable of a top underwater speed of 65 km/h. How fast would a 1/20 scale model of the submarine have to be moved through a testing pool filled with seawater for the forces on the submarine and model to be dimensionally similar?

\[ \text{Submarine: } \frac{V_{\text{sub}}}{V_{\text{model}}} = \frac{T_{\text{sub}}}{T_{\text{model}}} \]

(A) 0.90 m/s
(B) 18 m/s
(C) 180 m/s
(D) 360 m/s

\[ \Rightarrow V_{\text{model}} = \frac{0.90 \text{ m/s}}{20} = 0.045 \text{ m/s} \]

Drag

\[ F_D = \frac{1}{2} C_D \rho V^2 \]

Typically these problems will be "plug and chug"

Drag Coefficients for Spheres and Circular Flat Disks

\[ C_D = \frac{1}{2} \text{ for } \frac{V}{W} = 0 \]

\[ C_D = \frac{1}{2} \frac{V}{W} \text{ for } \frac{V}{W} < 1 \]